ESD-TR-67-626 MTR-540



# MOTION PARAMETERS FROM SCATTERING MATRIX MEASUREMENTS



MAY 1968



J. F. A. Ormsby
W. T. Payne

Work Performed for

ADVANCED RESEARCH PROJECTS AGENCY

Contract Administered by

DEVELOPMENT ENGINEERING DIVISION
DIRECTORATE OF PLANNING AND TECHNOLOGY
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Sponsored by
Advanced Research Projects Agency
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Project 8051
Prepared by

THE MITRE CORPORATION
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## FOREWORD

The work reported in this document was performed by The MITRE Corporation, Bedford, Massachusetts, for Advanced Research Projects Agency; the contract was monitored by the Directorate of Planning and Technology, Electronic Systems Division, Air Force Systems Command, under Contract AF 19(628)-5165.

## REVIEW AND APPROVAL



A. P. TRUNFIO
Project Officer
Development Engineering Division
Directorate of Planning & Technology

# ABSTRACT

Previous efforts to derive body motion parameters from monostatic scattering matrix data have been limited by mathematical complexity. An account of these efforts is given. Then a proposed method is described for the case of precession, which allows the simplicity of a geometric approach and avoids approximation in its concept.

This method is formulated as a minimization problem with only three of the precession parameters as the arguments. The basic idea is to find a cone on which the spacing of successive positions of the body axis shall be the dynamically correct spacing. Then the two remaining parameters are easily evaluated.

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## SECTION I

## INTRODUCTION

In the course of seeking methods of deriving motion parameters from scattering matrix data, a number of methods have been considered and reported earlier by the authors in a number of internal MITRE reports. These methods appear to be limited in practice either by computational complexity or by the need for approximation.

In view of such difficulties, it is expedient to consider, at least initially, a method which allows the simplicities of a geometric approach and conceptually avoids approximation. Since it is a monostatic method without approximation, it requires the assumption of a motion model which in this case is precession for objects of rotational symmetry. The formulation given here, however, can be made in terms of just three motion precession parameters. In this work, the assumption is made, of course, that such an object depolarizes at the operating frequency to an extent which allows the determination of the orientation of the body (electrical) axis in a plane normal to the radar line of sight. It is to be noted also that a monostatic method for determining the body axis orientation using any type of observation data for a body of axial symmetry yields ambiguities.

Before discussing the method itself, it is useful to put into perspective the general problem in terms of the motion model, observations, unknowns, and analytic formulation. Thus, the paper divides into two parts with background and alternatives first discussed followed by an exposition of the proposed method.

Finally, it is to be noted that, although we deal with one motion technique applied to scattering matrix data, the use of phase and polarization information in radar returns has found widespread application in work at MITRE, benefiting methods in motion and shape determination both for long and short pulse data on whole object or scattering center descriptions [1, 2]. One author (O) has applied the scattering matrix to wedge parameters in other internal MITRE reports.

## SECTION II

## THE PROBLEM

Using an earth based radar, observations are made of an object in orbit possibly undergoing additional rotation movement about its center of mass. We illustrate the three motions in Figure 1.

In general all three motions act to determine the object's orientation with respect to the radar so that this target-sensor orientation affects the various possible sensor measurements. In this paper we deal with the source data in the form of the polarization scattering matrix from a monostatic radar. Written in a circular polarization basis, this matrix can be represented for backscatter from a reciprocal scatterer as:

$$S_{\odot} = \begin{pmatrix} \alpha\, e^{i2\xi} & \nu\, e^{i2\Psi_F} \\ \nu e^{-i2\Psi_F} & \beta e^{-i2\xi} \end{pmatrix} \tag{1}$$

where $\alpha$, $\beta$, $\nu$ are complex, $\xi$ represents body orientation about the line of sight and, $\Psi_F$ is the Faraday rotation. [3, 4, 5]. For an object with a plane of symmetry containing the line of sight, $\alpha = \beta$. Use of monostatic data implies both a $\pi$ and $\pi/2$ ambiguity in $\xi$ for axially symmetric bodies. Since the sense of the body axis vector is not of interest here, the $\pi$ ambiguity need not be dealt with. The process of obtaining a solution provides a means

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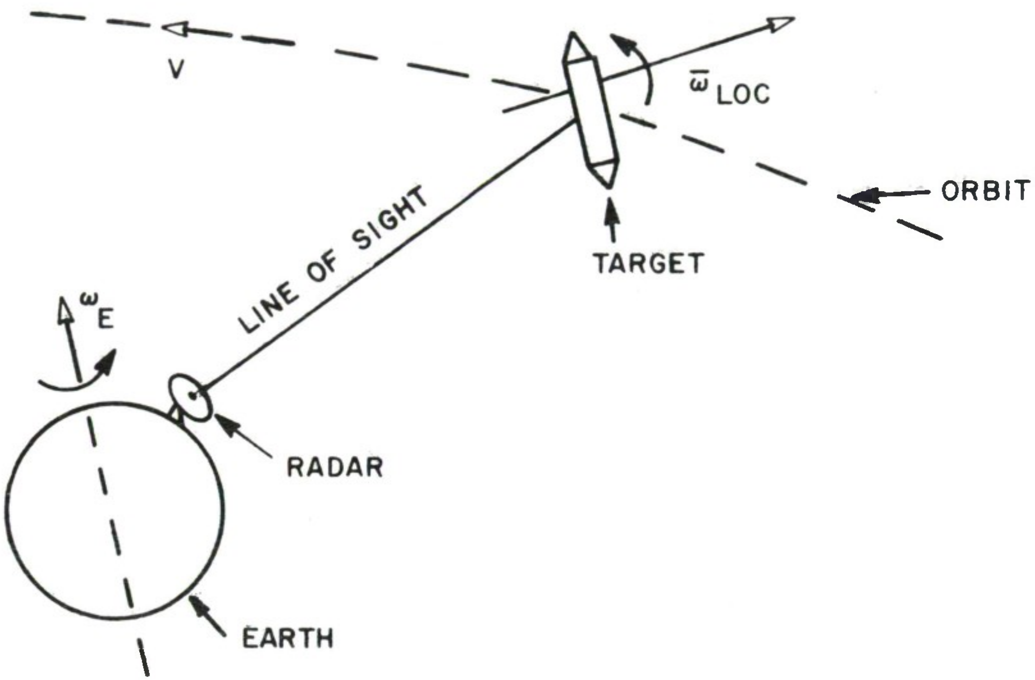


MOTION COMPONENTS:

$\overline{\omega}_t$ = EARTH ANGULAR VELOCITY (ROTATION RATE)

V = ORBIT VELOCITY

$\overline{\omega}_{LOC}$ = ANGULAR VELOCITY OF TARGET ABOUT ITS C.G.

Figure 1. General Motion System

in the monostatic case for resolving the $\pi/2$ ambiguity. The condition of sufficient depolarization is imposed to have $\alpha$ measurable. This latter condition is enhanced at operating frequencies near resonance. As the frequency increases with geometrical optics applying, all targets become sphere-like with $\alpha \to 0$. Finally the assumption of backscatterer is also an approximation, good for many motion environments.

We can consider the motion problem using scattering matrix data in terms of two planes designated the viewing plane and the measuring plane as shown in Figure 2.

Considering all motion effects absorbed into the measurement of $\xi(t)$, we find

$$\gamma(t) = \tan \xi(t) = \frac{< \hat{y}_R(t), \hat{u}(t) >}{< \hat{x}_R(t), \hat{u}(t) >} \tag{2}$$

in which $<, >$ means scalar product. The projections are indicated in Equation (2) to be onto time varying vectors $\hat{y}_R(t)$ and $\hat{x}_R(t)$. This time dependence implies that a fixed-in-space inertial frame is to be used as a reference system.

Equation (2) may be recast into the following useful forms

$$< [\gamma(t) \hat{x}_R(t) - \hat{y}_R(t)], \hat{u}(t) > = < \bar{n}(t), \hat{u}(t) > = 0 \tag{3}$$

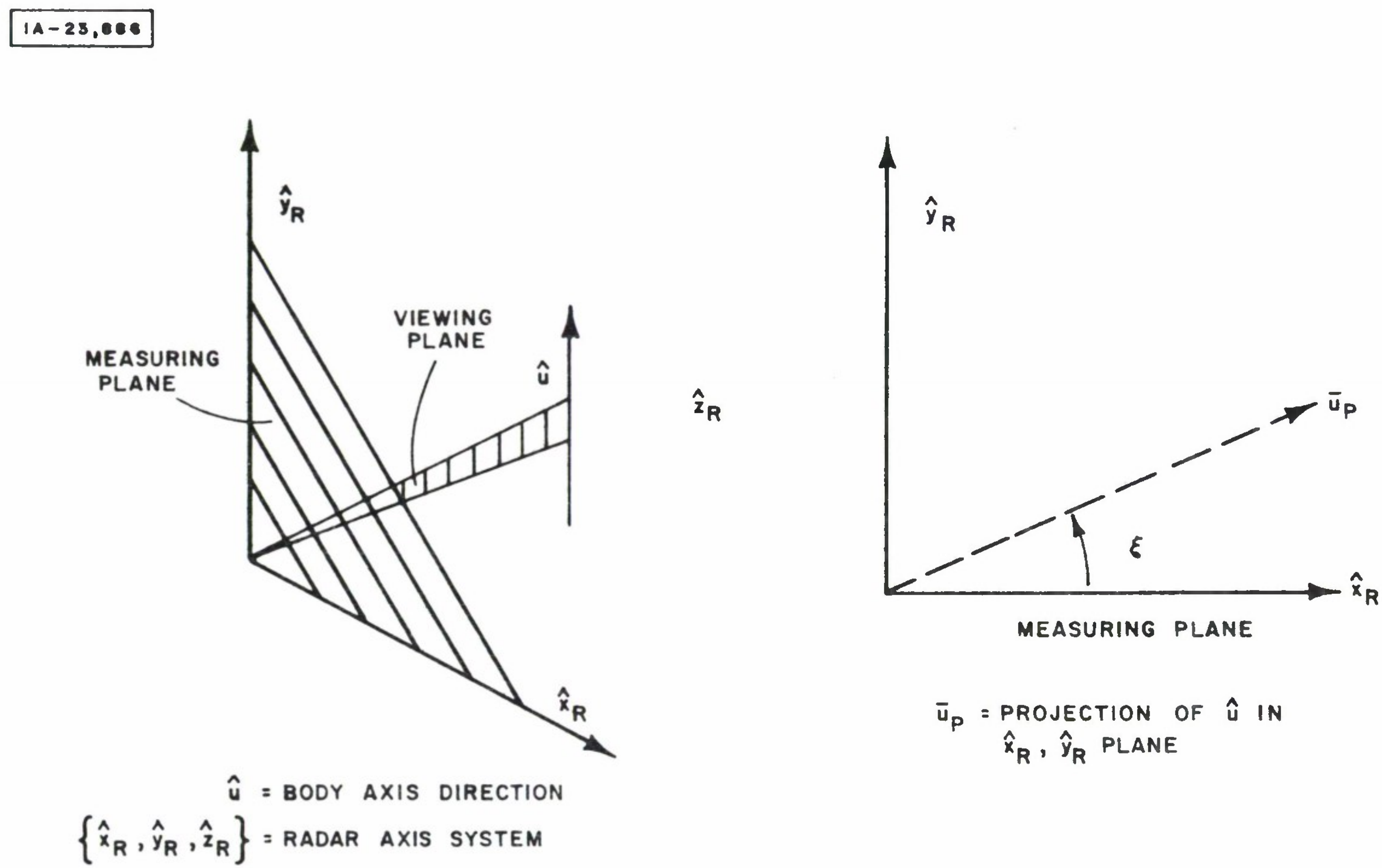


Figure 2. Motion Using Scattering Matrix Data

in which $\overline{n}(t)$ lies in the measuring plane and is normal to the viewing plane. With $\hat{u}$ taken as unrestricted in orientation and magnitude, Equation (3) is the equation of the viewing plane.

Before proceeding to more detailed analytical formulations, it is worthwhile to view the motion of the viewing plane as decomposed into infinitesimal rotations of $\hat{u}$ and $\hat{r}$ which are unit vectors in the direction of the body axis, $\overline{u}$, and line of sight, $\overline{r}$, respectively.

We illustrate the situation in Figure 3.

The infinitesimal rotation of the viewing plane can be given in terms of $\overline{R}$ as

$$\begin{aligned} \overline{dR} &= \overline{R} \times \overline{d\Omega} = \hat{r} \times \overline{d\Omega} + \hat{u} \times \overline{d\Omega} \\ &= \overline{dr} + \overline{du} \end{aligned} \tag{4}$$

Since any component of $\overline{d\Omega}$ normal to the viewing plane keeps this plane invariant in the $\hat{x}$, $\hat{y}$, $\hat{z}$ space, we can consider that $\overline{d\Omega}$ effectively lies in the viewing plane and gives the following decomposition

$$\overline{d\Omega} = \underset{(\text{along } \hat{r})}{\overline{d\Omega}_r} + \underset{(\text{along } \hat{u})}{\overline{d\Omega}_u}$$

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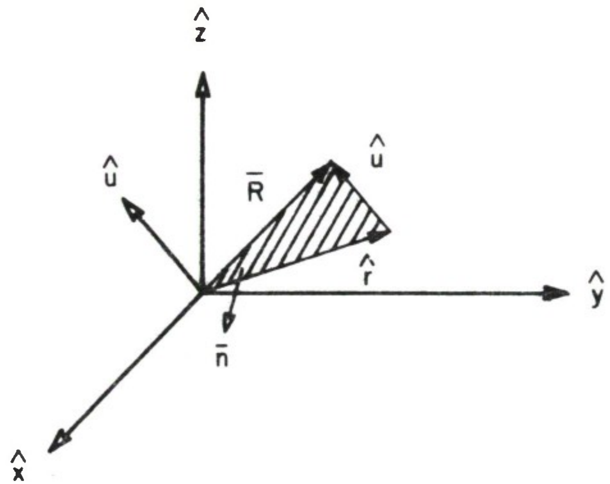


$\{\hat{x}, \hat{y}, \hat{z}\}$ = INERTIAL FRAME

$\bar{R} = \hat{r} + \hat{u}$

Figure 3. Viewing Plane Rotation Decomposition

Thus $\overline{d\Omega}_u$ determines the direction of $\hat{u}$ and

$$\overline{dR} = \underset{\substack{\text{(motion of } \hat{r} \\ \text{about } \hat{u})}}{\hat{r} \times \overline{d\Omega}_u} + \underset{\substack{\text{(motion of } \hat{u} \\ \text{about } \hat{r})}}{\hat{u} \times \overline{d\Omega}_r} \tag{5}$$

since

$$\hat{r} \times \overline{d\Omega}_r = \hat{u} \times \overline{d\Omega}_u = 0 \ .$$

For example with $\hat{u}$ fixed in inertial space,

$$\overline{du} = \hat{u} \times \overline{d\Omega} = \hat{u} \times \overline{d\Omega}_r = 0$$

so that $\hat{u}$ is parallel to $\overline{d\Omega}$, ($\overline{d\Omega} = \overline{d\Omega}_u$). Similarly for $\bar{r}$ fixed in inertial space, $\hat{r}$ is parallel to $\overline{d\Omega}$ ($\overline{d\Omega} = \overline{d\Omega}_r$). In both cases $\overline{d\Omega}$ has no component normal to the viewing plane.

With $\xi(t)$, the angular position of $\hat{u}$ about the line of sight direction $\hat{r}$, then $d\xi$ gives $|\overline{d\Omega}_r|$ .

If the conceptual model just outlined is viewed in terms of a possible approximation method for deriving the motion, not requiring the assumption of any motion model, then to give $\overline{d\Omega}_u = \overline{d\Omega} - \overline{d\Omega}_r$ and so the direction of $\hat{u}$, we need to measure or approximate $\hat{r}$, $\overline{d\Omega}$, and $\overline{d\Omega}_r$ such that

1) $\hat{r} = \hat{r}\,(\hat{\theta}, \hat{\phi}, \omega_E, \lambda, \eta)$ is determined

where

$\hat{\theta}$ = radar elevation

$\hat{\phi}$ = radar azimuth

$\omega_E$ = earth rotation rate

$\lambda$ = site longitude

$\eta$ = site latitude

2) $\overline{d\Omega}_r = |d\xi|\,\hat{r}$

3) $\overline{d\Omega} = |\overline{d\Omega}|\,\hat{i}_\Omega$ ; $\hat{i}_\Omega$ = unit vector in $\overline{d\Omega}$ direction

with

$$|d\Omega| = \left|\cos^{-1}\left\{\lim_{\Delta t \to 0} < \hat{n}(t+\Delta t), \hat{n}(t)> \right\}\right|$$

$$\hat{i}_\Omega = \lim_{\Delta t \to 0}\left[\frac{\hat{n}(t+\Delta t) \times \hat{n}(t)}{|\hat{n}(t+\Delta t) \times \hat{n}(t)|}\right]$$

It is to be noted that the restriction to a monostatic radar is most severe since at best, with no approximation, it requires the assumption of a motion model.

Returning to the formulation via the scattering matrix, the restriction to a motion model can be reasonably generalized to precession. We picture this in Figure 4 in terms of the inertial frame and an angular momentum axis system.

Taking account of the inertial reference and angular momentum systems, Equation (3) can be written as,

$$\sum_{j=1}^{3} \sum_{i=1}^{3} [\gamma(t)\ a_{1i}(t) - a_{2i}(t)]\ b_{ij}\ \tilde{u}_j(t) = 0 \qquad (6)$$

where $\gamma(t) = \tan \xi(t)$ is measurable,

$\tilde{u}_j(t) = \tilde{u}_j(\theta, \phi, \dot{\phi}_o, t)$; $j = 1, 2, 3$, are unknown coordinates of $\hat{u}$ in the $(\tilde{x}, \tilde{y}, \tilde{z})$ system and $\phi_o$ the initial value of $\phi$,

$a_{ij} = a_{ij}(\omega_E, \lambda, \eta, \hat{\theta}(t), \hat{\phi}(t))$; $i, j = 1, 2, 3$ are computable coefficients of the transformation from radar to inertial reference coordinates,

$b_{ij} = b_{ij}(\beta, \alpha)$; $i, j = 1, 2, 3$, are unknown coefficients of the transformation from angular momentum to inertial reference coordinates.

Formally at least Equation (6) holds for motion with torque in which $\beta = \beta(t)$, $\alpha = \alpha(t)$, $\theta = \theta(t)$, $\dot{\phi} = \dot{\phi}(t)$ and if spin is

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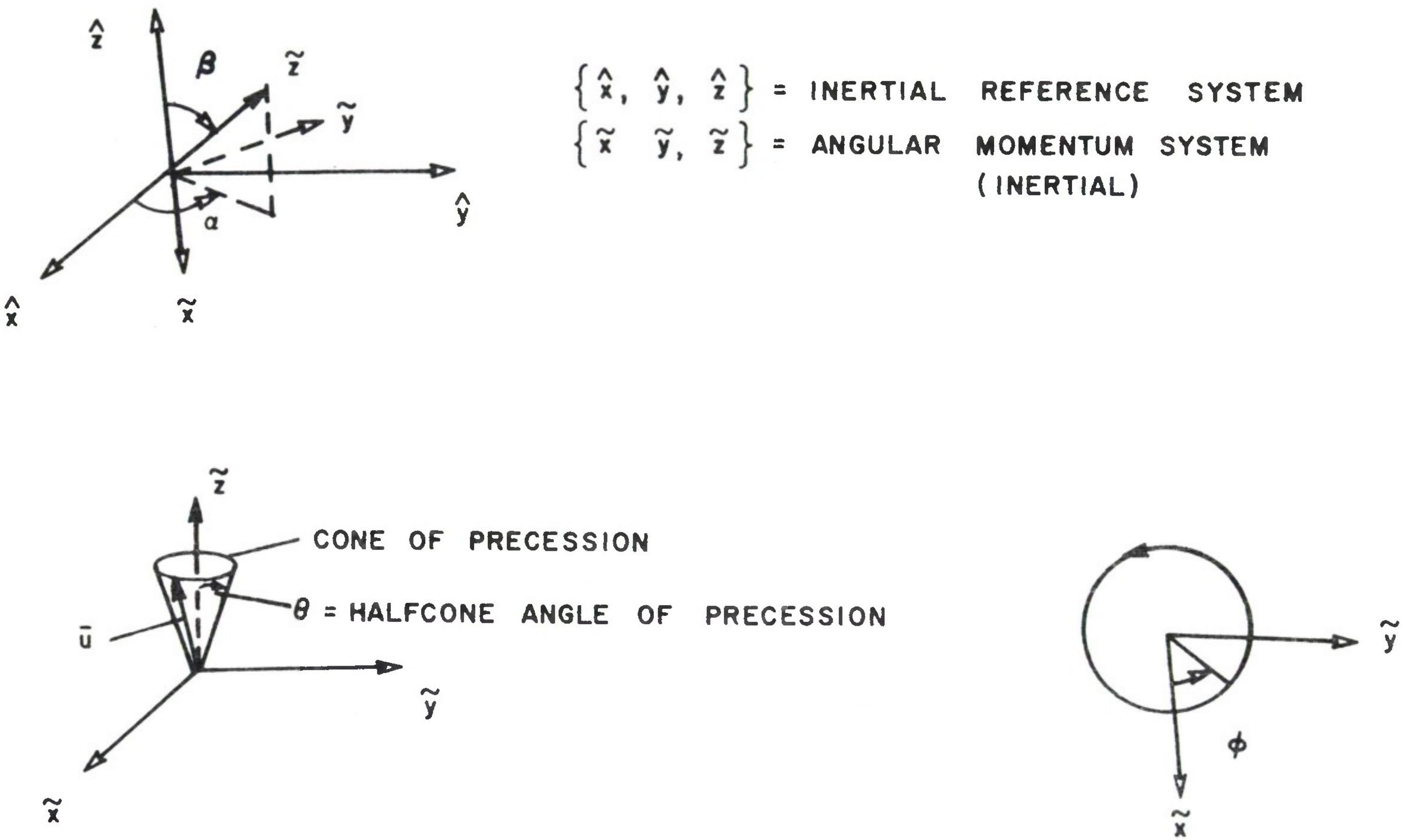


Figure 4. General Precessional Treatment

observable $\dot{\Psi} = \dot{\Psi}(t)$ where in each case general time functions are applicable. With torque free motion $\beta$ = constant and $\alpha$ = constant and $\theta(t)$, $\dot{\phi}(t)$, $\dot{\Psi}(t)$ can either be general (e.g. the case of an asymmetric top) or, for the further specification to precession, $\theta$, $\dot{\phi}$, and $\dot{\Psi}$ are constants. The formulation of $\hat{u}(t)$ relative to $\beta$, $\alpha$, $\theta$, $\dot{\phi}$, $\phi_o$ and $\dot{\Psi}$ is straightforward in this case. That is, $\hat{u}$ depends on six fixed parameters and five when, as here, spin is assumed unobservable. When general time functions are assumed, expansion representations can increase the number of unknown parameters to an unmanageable extent.

Thus, Equation (6) can be viewed as a system, with time, of space curves in 5-dimensional space of the unknown precession parameters $(\beta, \alpha, \theta, \phi_o, \dot{\phi})$. Then the equation is applied at $t = t_k$; $k \geq 5$ for solution of the parameters. This system is non-linear thus admitting a family of solutions with restrictions imposed by the allowable range of the parameter values.

Generally this is a most impractical approach to solution. Some help may be gained by measuring at times when $\gamma(t) = 0$ in order to simplify. In addition a formulation in terms of Cayley-Klein parameters and complex direction numbers can provide some compactness at least formally. We illustrate this by giving Equation (2) in detail in both conventional and Cayley-Klein form.

$$\gamma(t) = \left\{ \frac{\sum_{j=1}^{3} \sum_{i=1}^{3} a_{2i}(\beta_1, \beta_2, \beta_3, t)\, b_{ij}(\beta, \alpha)\, \tilde{u}_j\, (\theta, \phi_o, \dot{\phi}, t)}{\sum_{j=1}^{3} \sum_{i=1}^{3} a_{1i}(\beta_1, \beta_2, \beta_3, t)\, b_{ij}(\beta, \alpha)\, \tilde{u}_j\, (\theta, \phi_o, \dot{\phi}, t)} \right\} \tag{7a}$$

$$\xi(t) = \text{ARG} \left[ \frac{(\mu_a \mu_b^* + \nu_a \nu_b^*)\, (\cot \theta/2\, e^{i\dot{\phi}t}) + (-\mu_a \nu_b + \nu_a \mu_b)}{(-\nu_a^* \mu_b^* + \mu_a^* \nu_b^*)\, (\cot \theta/2\, e^{i\dot{\phi}t}) + (\nu_a^* \nu_b + \mu_a^* \mu_b)} \right] \tag{7b}$$

where $\cot \theta/2\, e^{i\dot{\phi}t}$ is the complex direction of $\hat{u}$, initial $\phi_o = 0$, and the Cayley-Klein parameters are

$$\mu_a = \cos \frac{\beta_1}{2}\, e^{(-i/2)\, (\beta_2 + \beta_3)}; \qquad \nu_a = \sin \frac{\beta_1}{2} e^{(i/2)\, (\beta_2 - \beta_3)}$$

$$\mu_b = \cos \frac{\beta}{2}\, e^{(-i/2)\, (\alpha + \phi_o)}; \qquad \nu_b = \sin \frac{\beta}{2}^{(i/2)\, (\alpha - \phi_o)}$$

with

$\beta_1, \beta_2, \beta_3$ = Euler angles relating inertial system to radar system,

$\beta, \alpha, \phi_o$ = Euler angles relating inertial system to momentum system,

and $(\quad)^*$ the conjugate of $(\quad)$.

The coefficients of $\cos \theta/2\, e^{i\dot{\phi}t}$ ($\theta$ and $\dot{\phi}t$ are the spherical angles of the direction) in the linear fractional transformations of (7b) represent the combined effect of transforming from the radar to the inertial system via

$$\begin{pmatrix} \mu_a & \nu_a \\ -\nu_a^* & \mu_a^* \end{pmatrix}$$

and from the momentum to inertial system via

$$\begin{pmatrix} \mu_b & \nu_b \\ -\nu_b^* & \mu_b^* \end{pmatrix}$$

by matrix multiplication. The parallelism between (7a) and (7b) is complete if, as is possible, we take in (7a)

$$b_{ij} = b_{ij}(\beta, \alpha, \phi_o)$$

and

$$\tilde{u}_j = \tilde{u}_j(\theta, 0, \dot{\phi}, t)$$

An alternate phrasing of (7a) sometimes considered can be written as

$$\gamma(t) = \left\{ \frac{\sum_{j=1}^{3} C_{2j}(\beta_1', \beta_2', \beta_3')\, \tilde{u}_j(\theta, \phi_o, \dot{\phi}, t)}{\sum_{j=1}^{3} C_{1j}(\beta_1', \beta_2', \beta_3')\, \tilde{u}_j(\theta, \phi_o, \dot{\phi}, t)} \right\} \tag{8}$$

where

$$C_{kj} = \sum_{i=1}^{3} a_{ki}(\beta_1, \beta_2, \beta_3, t)\, b_{ij}(\beta,\alpha) = \sum_{i=1}^{3} a_{ki}(\omega_E, \lambda, \eta, \hat{\theta}(t), \hat{\phi}(t))\, b_{ij}(\beta,\alpha)$$

$$k = 1, 2$$

so that

$$\beta_s' = \beta_s' (\beta, \alpha, t); \; s = 1, 2, 3$$

Also,

$$\cos A(t) = \sum_{j=1}^{3} \sum_{i=1}^{3} a_{3i} b_{ij} (\beta, \alpha) \tilde{u}_j$$

where $A(t)$ is the aspect angle between $\hat{u}$ and $\hat{r}$. However, this relating of the momentum to radar systems directly hence of $\hat{u}$ to the radar in terms of aspect angle $A(t)$ is of little consequence since the parameters $\beta$, $\alpha$ must eventually be exposed for a proper solution.

More importantly, all approaches which attempt to solve for the precession parameters via this system of non-linear equations are at best arduous. Rather if this equation, $\langle \bar{n}, \hat{u} \rangle = 0$ is considered as the equation of a plane (viewing) in combination with a priori knowledge of precession mechanics then an alternate and greatly simplified approach emerges.

However, before passing on to the details of the method, a few remarks concerning particular cases are now appropriate.

Case (1) $\hat{u}$ fixed in space (stabilized inertially).

Then the intersection of two viewing planes at separate times provides $\hat{u}$ as illustrated in Figure 5.

Also with very slow motion in inertial space, with no motion model assumed, the above method can serve as an approximate solution.

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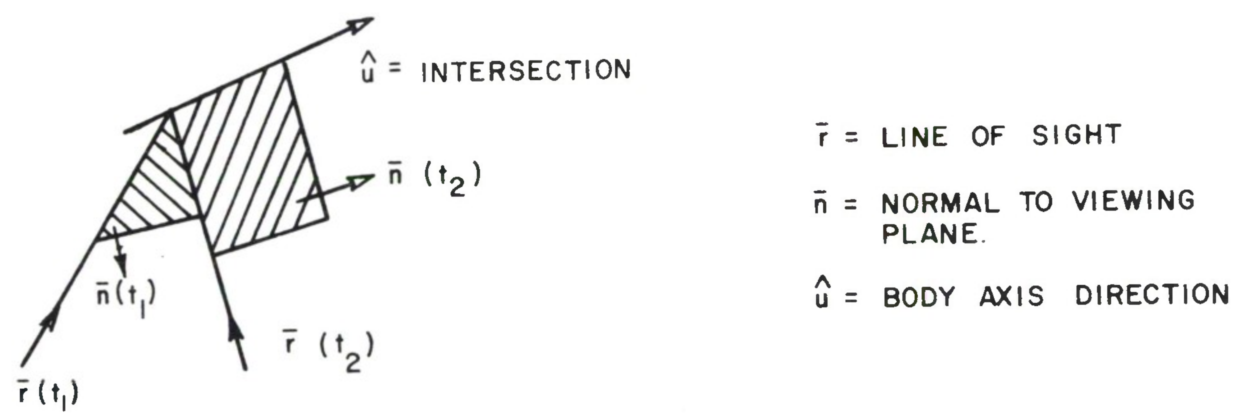



Figure 5. Intersecting Viewing Planes

With $\hat{u}$ determined the angular momentum unit vector direction $\hat{m}$ is then given by the eigenvector of $U^T U$ for the minimum eigenvalue where

$$U = (\Delta u_{ij} = (u_{i+1,j} - u_{i,j}); \; j = 1, 2, 3; \; i = 1, 2, \ldots, N$$

and $u_{ij}$ are the components of $\hat{u}(t_i)$ in inertial coordinates.

Case (2) If the local motion, $\bar{\omega}_{LOC}$, is very rapid compared to other motions, then with $t_j - t_{j-1} = T$, where T is the precession period, $\hat{u}(t_j)$ is effectively stationary and is given by the eigenvector of $\varkappa^T \varkappa$ for the minimum eigenvalue where, using components of $\hat{n}$,

$$\varkappa = (n_{ij}); \; j = 1, 2, 3; \; i = 1, 2, \ldots, N + 1$$

A test on the determination of T can be made in terms of the singularity of $\varkappa$. After $\hat{u}$ is obtained then $\hat{m}$ can be obtained from $[u(t) + u(t + T/2)]$. With all other precession parameters obtained a recalculation of T, using the formulation of $\hat{u}(t)$ in terms of $\theta$, $\phi_o$, $\dot{\phi}$, can be made to adjust T and repeat the process for improved estimates.

## SECTION III

## THE METHOD

Briefly the method relies on the correct solution having equal values of actual or scaled azimuth spacing on the precession cone of the body axis orientation determined at successive pulse times that is on the linear relation between azimuth and time. The specification of body axis orientation is made as the intersection of the half cone of precession and a plane, the viewing plane, containing the line of sight and the body axis (the viewing plane also contains the projection of the body axis onto the measuring plane).

More particularly, an initial trial cone of precession is chosen by specifying with respect to the chosen inertial system the azimuth ($\alpha_c$) and polar angle ($\beta_c$) of the cone axis as well as its half angle of precession ($\theta_c$). With $\alpha_c$, $\beta_c$, $\theta_c$ given, the intersection of the cone with a sequence of say N viewing planes provides at most a sequence of 2N body axis directions denoted by $\{\hat{u}_i\}_{i=1}^{2N}$ since each plane and the cone intersect along two direction lines.

Using unperturbed data measured at equal time increments or appropriately scaled one of the consistent subsequences of $\{\hat{u}_i\}_{i=1}^{N}$ will result in equal azimuth spacing about the cone axis if the trial cone parameters $\alpha_c$, $\beta_c$, $\theta_c$ are correct. Equivalently stated, such a sequence of azimuth differences will have zero variance. The parameters $\alpha_c$, $\beta_c$, and $\theta_c$ on which the azimuth values $\{\phi_i\}$ and so the variance depend are adjusted to the optimum values for minimum variance which is zero in the unperturbed case. Various functional minimization procedures are possible. It is possible to begin with a form of gradient method known as direct search, then as required by a reduced convergence rate to switch to another calculation, say Newton-Raphson, near the vicinity of the minimum.

The testing of the method can be done with the aid of a radar motion simulation program developed in mid-1964. This simulator characterizes motion in terms of the earth, orbit and local (about object c.m.) motion. The latter allows for general torque free motion or various stabilized motions. An acquisition capability for any number of sites and objects is also included. It provides a scattering matrix radar model in linear or circular polarization with conversion, and with data either unperturbed or perturbed by Faraday rotation, signal level and phase variation with range and various noise in amplitude and phase. In addition a model is included for handling the bistatic case. In the simulator the angle $\xi(t)$ is obtained as an Euler angle relating a body axis system to the radar system of coordinates.

## SECTION IV

## FORMULATION

From scattering matrix data the $i^{th}$ azimuth, $\xi_i$, of the body axis projection on the plane normal to the line of sight allows for a determination of the orientation of the $i^{th}$ viewing plane in terms of its unit normal $\hat{n}_i$ as

$$\hat{n}_i = \sin \xi_i \, \hat{x}_R - \cos \xi_i \, \hat{y}_R$$

where $\hat{x}_R$ and $\hat{y}_R$ are unit coordinate vectors in the radar system. Written in the chosen inertial frame, the components of $\hat{n}_i$ are denoted by $n_{i1}$, $n_{i2}$, $n_{i3}$. The basic geometry is reviewed in Figure 6.

In order to maintain a consistent choice of test half cone, $\langle \hat{u}, \hat{m} \rangle$ is taken, say, always positive where $\hat{m}$ is the unit vector of the trial cone axis direction (along the angular momentum line). In addition, it is useful in choosing consistent precession angle values, $\phi$, and in setting up tests to deal with noisy data to take $\langle \hat{n}, \hat{m} \rangle > 0$ that is

$$\sum_{j=1}^{3} n_{ij} m_j = n_{i1} \sin \beta_c \cos \alpha_c + n_{i2} \sin \beta_c \sin \alpha_c + n_{i3} \cos \beta_c$$

$$= \tilde{n}_{i3} > 0$$

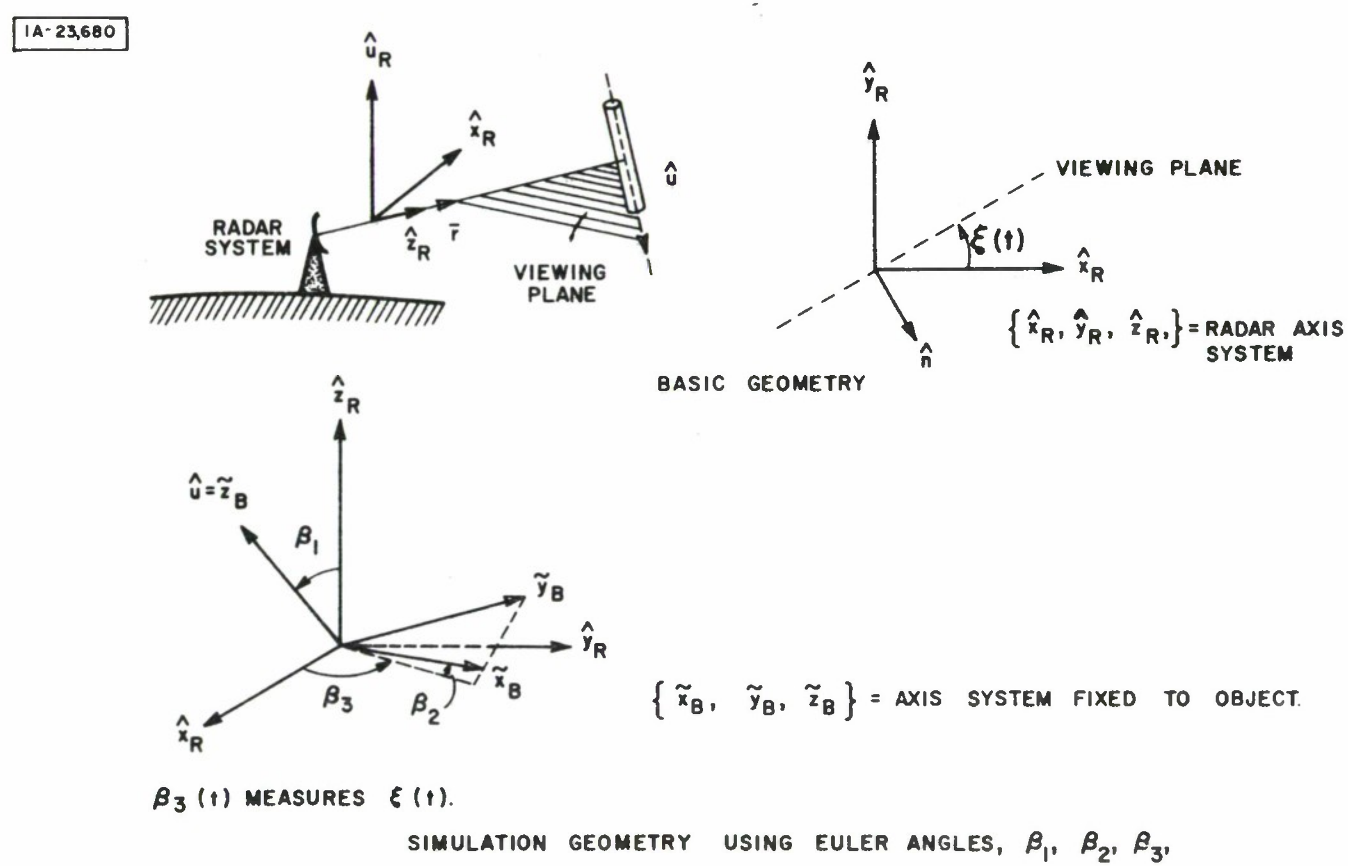



Figure 6. Basic Geometry and Simulation

where

$m_j$; j = 1, 2, 3, are the coordinates of $\hat{m}$ in the inertial system

$\tilde{n}_{ij}$; j = 1, 2, 3, are the coordinates of $\hat{n}_i$ in the test cone axis system $(\hat{x}, \hat{y}, \hat{z})$

$\hat{z} = \hat{m}$ is related to the inertial system by the polar and azimuth angles $\beta_c$ and $\alpha_c$ respectively, and

$$0 \le \beta_c < \pi/2; \; 0 \le \alpha_c < 2\pi \; .$$

If

$$\sum_{j=1}^{3} n_{ij} m_j < 0 \quad ,$$

the signs of $n_{ij}$; j = 1, 2, 3, are reversed.

In addition to $\alpha_c$ and $\beta_c$ the trial half cone is then fully specified by assignment of $\theta_c$, the half angle of precession with $0 < \theta_c \le \pi/2$.

With noïsy data, the intersection of all viewing planes with the correct cone is not assured. To allow intersection of the test cone with most of the measured viewing planes, if possible, $\theta_c$ is chosen so as to exclude only a small percentage, say $\mu$ of these planes. We may phrase this situation as follows; let

$$\Omega(\theta_c) = \{i; \tilde{n}_{i3} \le \sin \theta_c\}$$

having measure

$$0 \leq \mu_{\theta_c} \leq 1 .$$

Further, certain viewing planes which are preserved by the above selection process may be near tangency. Of the viewing planes chosen we select the set of i values denoted by $\Omega(\rho, \theta_c)$ where

$$\Omega(\rho, \theta_c) \subseteq \Omega(\theta_c)$$

such that,

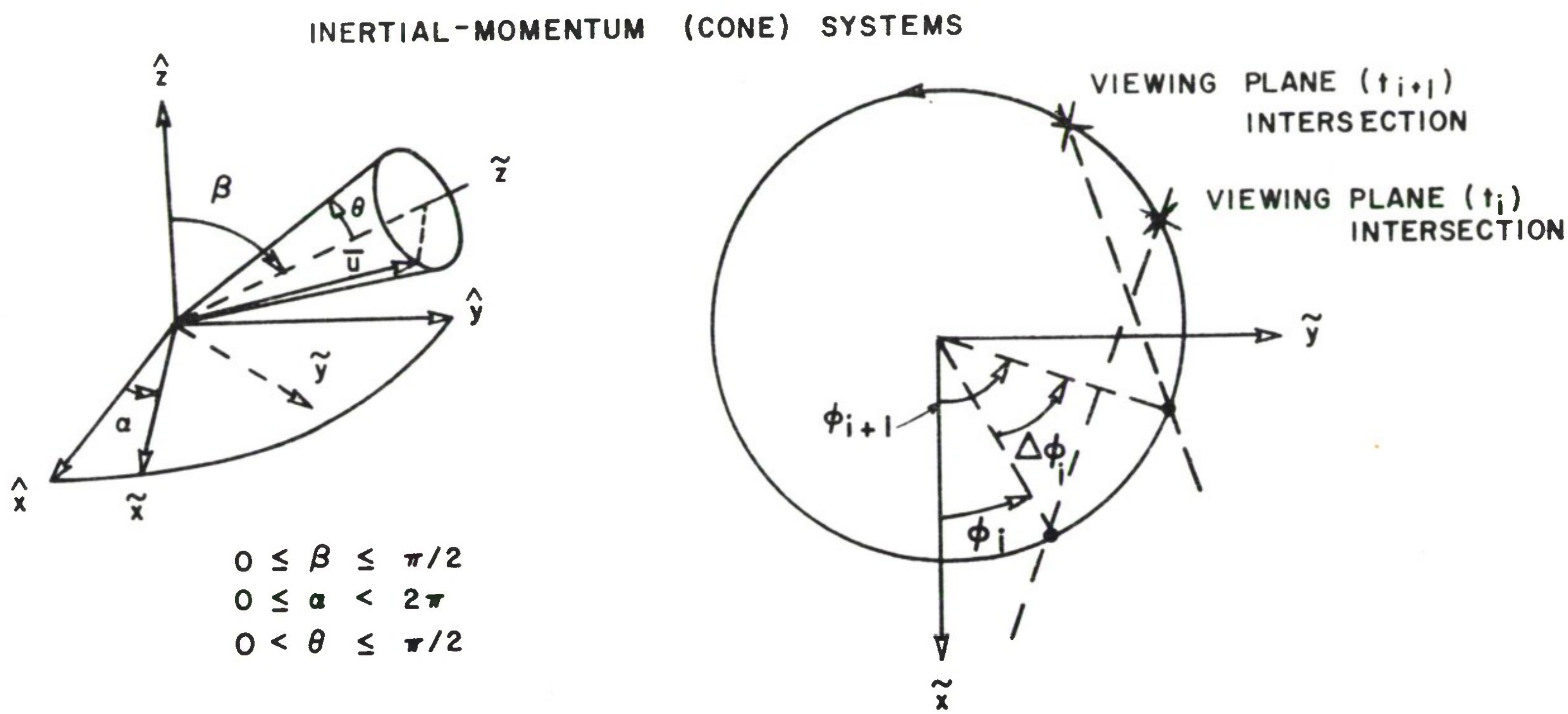
$$\Omega(\rho, \theta_c) = \{i;\ i\epsilon\Omega(\theta_c),\ \tilde{n}_{i3} \leq \rho \sin \theta_c\}$$

Thus, $\mu_{\theta_c}$ and $\rho$ are chosen to be less than but near 1; for example $\mu_{\theta_c} = .99$ and $\rho = .95$.

In order to visualize the analytic formulation more clearly, we show the geometry in Figure 7.

As noted, for unequal interpulse times, the $\Delta\phi_i$ values are simply scaled for equal values $\Delta\tilde{\phi}_i$ using for each i

$$\Delta\tilde{\phi}_i = \Delta\phi_i \left(\frac{\Delta t_i}{\Delta t_{REF}}\right)$$




$\beta = \beta_c$, $\alpha = \alpha_c$, $\theta = \theta_c$; TRIAL CONE VALUES

VIEWING PLANE ($t_i$) INTERSECTS CONE ALONG

$\bar{u}_i$ WITH AZIMUTH $\phi_i$

Figure 7. Geometry of the Motion Method

Using then the two equations of the viewing plane and half cone in test cone coordinates we obtain

$$\langle \hat{n}_i, \hat{x} \rangle = \tilde{n}_{i1} (\sin\theta \cos\phi) + \tilde{n}_{i2} (\sin\theta \sin\phi) + \tilde{n}_{i3} \cos\theta = 0 \tag{9}$$

$$\theta = \theta_c \tag{10}$$

With $\theta_c < \pi/2$, this establishes (with $\hat{\eta}$ are already chosen) consistently the test half cone. The sense of $\hat{\eta}$ is set by constraints on $\alpha_c$ and $\beta_c$.

Denoting the azimuth of the $i^{th}$ determination of $\hat{u}$ as $\phi_i$ in the axis system $(\tilde{x}, \tilde{y}, \tilde{z})$ attached to the test cone as previously described, we obtain as a function of $\alpha_c$, $\beta_c$, and $\theta_c$ only

$$\phi_i = \tan^{-1} \left( \frac{\tilde{n}_{i2}}{\tilde{n}_{i1}} \right) + \cos^{-1} \left( \frac{-\tilde{n}_{i3}}{(\tilde{n}_{i1}{}^2 + \tilde{n}_{i2}{}^2)^{\frac{1}{2}} \tan\theta_c} \right) \tag{11}$$

$$= f_i(\alpha_c, \beta_c,) + g_i (\alpha_c, \beta_c, \theta_c) \quad .$$

where

$$\tilde{n}_{ij} = \tilde{n}_{ij}(\alpha_c, \beta_c); \; j = 1, 2, 3 \quad .$$

The first term $f_i$ measures the azimuth of the vector $\hat{n}_i$ while $g_i$ measures the azimuth of $\hat{u}_i$ relative to $\hat{n}_i$ with both measurements in the $\tilde{x}$, $\tilde{y}$ plane of the trial cone frame.

Since, with $\tilde{n}_{i3} > 0$ ,

$$\left[\frac{-\tilde{n}_{i3}}{(\tilde{n}_{i1}^2 + \tilde{n}_{i2}^2)^{\frac{1}{2}} \tan \theta_c}\right] \leq 0 ,$$

then,

$$\pi/2 \leq g_i \leq 3\pi/2$$

which allows two solutions for $\hat{u}_i$ symmetric about $g_i = \pi$. We denote these solutions as $g_{i1}$ and $g_{i2}$ where the consistent selection of $f$ , $g$ and so $\phi$ values is based on sign changes in $\tilde{n}_{i3}$ , the possible monotonicity of $\phi$ , and consistent $(\Delta\phi)$ values.

$$\pi/2 \leq g_{i1} \leq \pi , \quad \pi \leq g_{i2} \leq 3\pi/2 .$$

Using

$$\phi_{ik} = f_i + g_{ik}; \; k = 1, 2,$$

we calculate

$$\Delta\phi = \frac{\phi_{jk} - \phi_{ik}}{j-i}$$

where,

$$j = \underset{i'}{\mathrm{MIN}} \{i'; \; i' > i\}$$

for all i allowed by the selection rules with k = 1, 2 and take j-i calculations for each i, j pair.

The azimuth relationships are shown in Figure 8.

Thus using the criterion of equal $\Delta\phi$ values, a function is constructed which for the proper g set and correct precession parameters is minimum.

Denoting the set of such differences as $\{\Delta\phi_{mk}\}_{m=1}^{M}$; k = 1, 2 then we calculate for k = 1, 2;

$$\overline{\Delta\phi}_k = \frac{1}{M} \sum_{m=1}^{M} \Delta\phi_{mk} \quad , \text{ the mean}$$

and

$$\sigma^2_{\Delta\phi_k} = \frac{1}{M} \sum_{m=1}^{M} (\Delta\phi_{mk} - \overline{\Delta\phi}_k)^2 \quad , \text{ the variance}$$

where

$$\sigma^2_{\Delta\phi_k} = \sigma^2_{\Delta\phi_k} (\alpha_c, \beta_c, \theta_c) \; .$$

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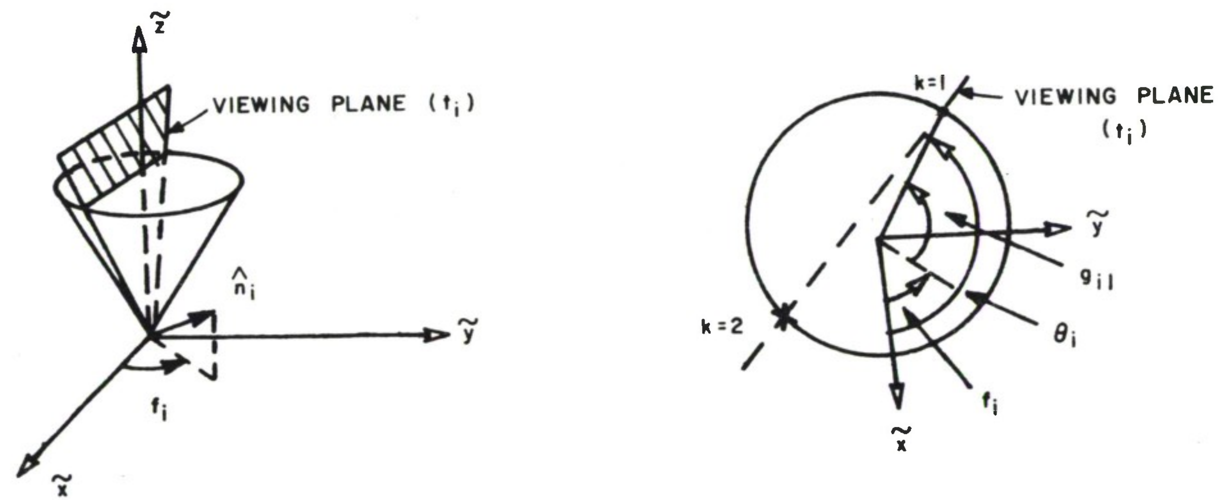


$\phi_{ik} = f_i + g_{ik}$ ; $f_i$ MEASURES AZIMUTH OF $\hat{n}_i$ IN $\tilde{x}, \tilde{y}$ PLANE

$g_{ik}$ MEASURE AZIMUTH OF $\hat{u}_i$ FROM $\hat{n}_i$ IN $\tilde{x}, \tilde{y}$, PLANE

$k = 1, 2$

Figure 8. Azimuth Relations

Notice, the formulation is derived in terms of just three precession parameters so that we seek

$$\beta_c^*, \alpha_c^*, \theta_c^*,$$

to give

$$\sigma^2_k (\beta_c^*, \alpha_c^*, \theta_c^*, ) = \underset{\beta_c, \alpha_c, \theta_c}{\mathrm{MIN}} \sigma^2_k [(\beta_c, \alpha_c, \theta_c)] \tag{12}$$

If we take $\theta_c' = \theta_c + \Delta\theta_c$ with $\frac{\Delta\theta_c}{\theta_c} \ll 1$ and, calculate $\sigma^2_{\Delta\phi_k} (\alpha_c, \beta_c, \theta_c')$, $k = 1, 2$ after applying the selection rules based on $\mu_{\theta_c}$ and $\rho$, and proceed similarly for

$$\sigma^2_{\Delta\phi_k} (\alpha_c', \beta_c, \theta_c) \text{ and } \sigma^2_{\Delta\phi_k} (\alpha_c, \beta_c', \theta_c)$$

also including in these two cases the proper initial adjustment of the sign of $\tilde{n}_{i3}$, then the function $\sigma^2$ can be altered by simultaneous changes in coordinates $(\alpha_c, \beta_c, \theta_c)$ along a direction approximating steepest descent. Such a method generating so called normal equations is often inefficient in computing. In these methods $\frac{\partial\sigma^2}{\partial(\ )}$ evaluated or approximated for ( ) equal to $\alpha_c$, $\beta_c$ or $\theta_c$ gives the new value directly as,

$$\frac{\partial\sigma^2}{\partial\alpha_c} \Delta\alpha_c + \frac{\partial\sigma^2}{\partial\beta_c} \Delta\beta_c + \frac{\partial\sigma^2}{\partial\theta_c} \Delta\theta_c + \sigma^2_{\Delta\phi_k} (\alpha_c, \beta_c, \theta_c)$$

$$k = 1, 2$$

The actual algorithm used initially will follow a method called direct search. Adaptation and revisions of this approach have been considered in internal MITRE reports. This method uses two basic approach moves called exploratory and pattern. Each pattern move follows a successful exploratory move by duplicating this movement along the direction of change established by that exploratory move.*

If an exploratory move cannot effect a decrease in the function to be minimized, here $\sigma^2$, the coordinate position is returned to that effected by the previous exploratory move. Then a new exploration is made which if unsuccessful results in a reduction of coordinate step size and another exploration etc. In each exploration the coordinate size is fixed and its sign is chosen to cause a function decrease if possible. Each coordinate is treated sequentially starting with the function value established by the previous coordinate change.

Each pattern move whether successful or not is followed by an exploratory move from a new base point upon success or the former base point on failure.

It appears that other useful additions to this process might check on the rate of change with respect to the coordinates, in the manner

*This is a variation of the usual approach of duplicating both the previous pattern and exploratory moves. For a sequence of successful pattern moves this could amount to duplicating the effect of all previous exploratory moves. A preferred approach would seem to be one which weights recent exploratory moves more favorably.

outlined above under the discussion of partial derivatives to establish initial coordinate step size ratios as well as to determine when to switch to another method if the step sizes are too small in the vicinity of solution.

In summary the motion method is as follows:

1. Select initial $\beta_c$, $\alpha_c$, $\theta_c$ values and compute $\sigma_k^2(\beta_c, \alpha_c, \theta_c)$ applying selection rules. Other calculations, such as intersecting planes, provide aid in choosing initial values.

2. Apply minimization procedure to give correct $\beta_c^*$, $\alpha_c^*$, $\theta_c^*$ values for k = 1, 2, using for example modified direct search and Newton-Raphson methods. Redundancy in the data with the correct k value can be used to improve estimates.

3. Establish $\dot{\phi}$, $\phi_o$ directly from $\phi_i$ values and the optimum set $\beta_c^*$, $\alpha_c^*$, $\theta_c^*$ .

Obvious extensions to the bistatic case, using the simulator again as a test mechanism, can be made. The testing procedure will emphasize unperturbed data to test the accuracy and usefulness of the basic method. This will be followed by tests with "noisy data" to evaluate the capability for practical operation.

## REFERENCES

1. Payne, W. T., "Analysis of the Polarization Scattering Matrix", The MITRE Corp., ESD-TR-67-627, March 27, 1968.

2. Tomljanovich, N. M., "A Different and More Direct Approach to the Inverse Edge Backscattering Problem", The MITRE Corp., ESD-TR-68-112, March 27, 1968.

3. Bickel, S. H., "Some Invariant Properties of the Polarization Scattering Matrix", Proc. IEEE, Vol. 53, No. 8, August 1965. Also as MTP-2, The MITRE Corp., July 1965.

4. Bickel, S. H. and Ormsby, J.F.A., "Error Analysis, Calibration, and the Polarization Scattering Matrix", Proc. IEEE, Vol. 53, No. 8, August 1965. Also as MTP-3, The MITRE Corp., July 1965.

5. Bickel, S. H. and Bates, R. H. T., "Effects of Magneto - Ionic Propagation on the Polarization Scattering Matrix", Proc. IEEE, Vol. 53, No. 8, August 1965. Also as MTP-4, The MITRE Corp., July 1965.

6. Hooke, R. and Jeeves, T.A., " 'Direct Search' to the Solution of Numerical and Statistical Problems", Assoc. for Computing Machinery, 8, 212 (1961).

7. Wood, C.F., "Application of 'Direct Search' to the Solution of Engineering Problems". Westinghouse Res. Lab. Scientific Paper No. 6-41210-1-P1(1960).

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13. ABSTRACT


Previous efforts to derive body motion parameters from monostatic scattering matrix data have been limited by mathematical complexity. An account of these efforts is given. Then a proposed method is described for the case of precession, which allows the simplicity of a geometric approach and avoids approximation in its concept.

This method is formulated as a minimization problem with only three of the precession parameters as the arguments. The basic idea is to find a cone on which the spacing of successive positions of the body axis shall be the dynamically correct spacing. Then the two remaining parameters are easily evaluated.


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